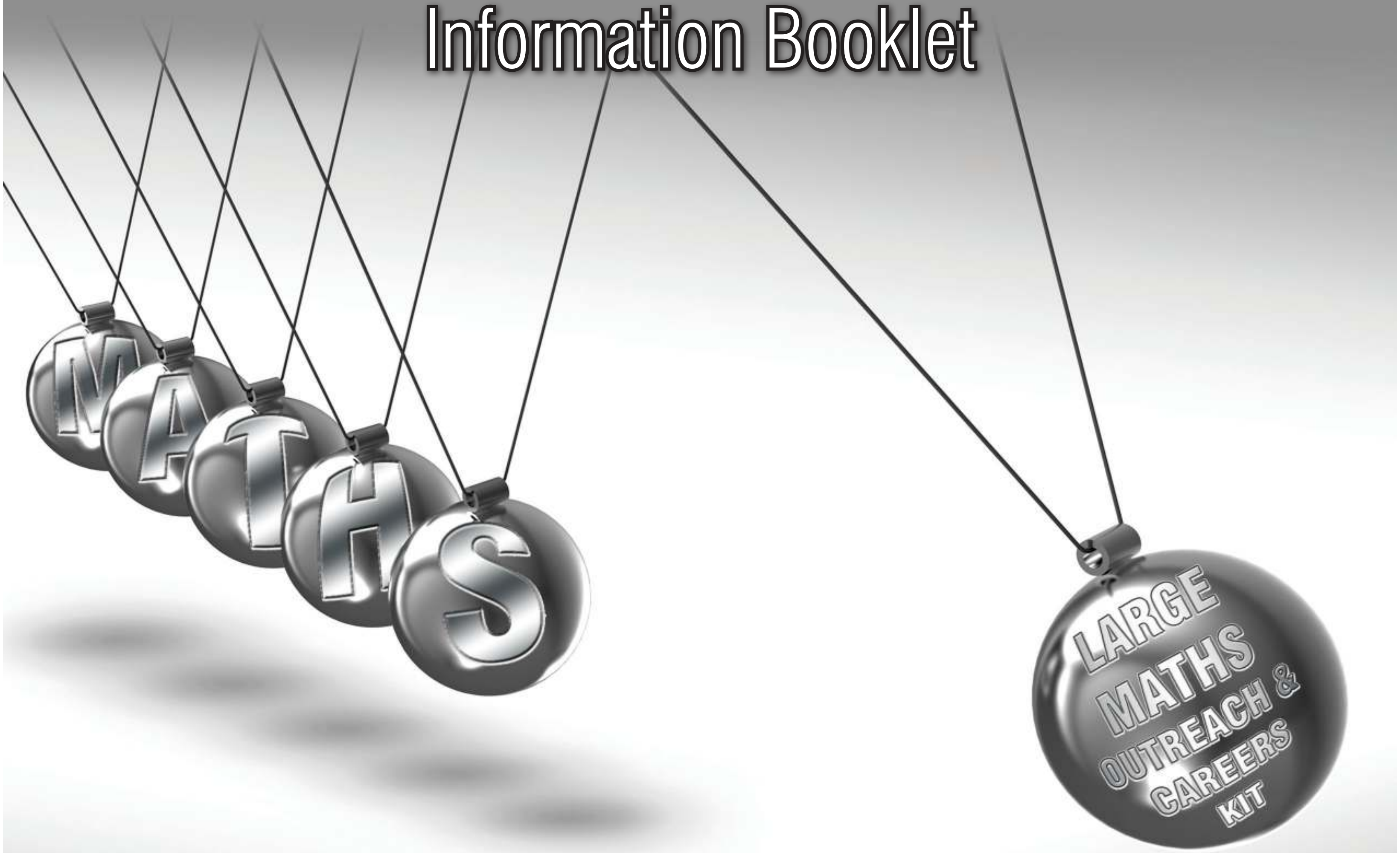
National HE STEM Programme

LARGE MATHS OUTREACH and CAREERS KIT

Information Booklet



maths careers endless opportunities

Institute of mathematics & its applications

Watch the training DVD to see all Large Maths Outreach & Careers Kit items in action.
Also visit www.mathscareers.org.uk

Large Maths Outreach and Careers Kit
Information Booklet

Welcome to the Large Maths outreach and Careers Kit!

The Large Maths Outreach and Careers Kit was developed by the Institute of Mathematics and its Applications (IMA) as part of the National HE STEM Programme. It was created following feedback from the HEI maths community who voiced that it was becoming more common for them to be asked to do something 'hands on' when doing outreach work with schools; particularly when they have a stall at an outreach event, careers fair, science fair or exhibition.

Two kits have been developed: a small briefcase sized kit which contains a number of small hands on activities, as well as this Large Maths Outreach and Careers Kit.

The Large Maths Outreach and Careers Kit consists of seven large hands on activities which could be a focus of a stall at a science fair or careers fair.

This booklet Contains:

- A tutorial DVD showing the kit contents in action.
- A series of exciting information sheets explaining some of the maths behind the activities in a simplified way - the aim is that similar ideas might help inform discussions with school students.
- A feedback form, a copy of which will be emailed to you if you have loaned one of the items.

The fact sheets on the Penrose Tiles were produced as part of the 2009 Royal Society Summer Exhibition. There are more fact sheets and more information to be found at: www.tilings.org.uk/shapes The fact sheet on the Aerofoil was produced by Dr Alison Hooper from the University of the West of England.

The Large Maths Outreach and Careers Kit was put together by the IMA using the wisdom, experience and suggestions of some of the most experienced maths outreach practitioners from HEI maths departments and maths education departments around England and Wales

If you have any further questions about this kit, please contact the Institute of Mathematics and its Applications via our website at www.ima.org.uk.

We hope you enjoy using the kit!

Important Note: Anyone borrowing the Large Maths Outreach and Careers Kit is responsible for the safe use of the items, for reading all instructions and preparing appropriate risk assessments should they be necessary. It is strongly advisable to practise using the items before taking them to an event or fair.

It is suggested that you download the 'Getting to grips with manual handling' booklet from www.hse.gov.uk to observe good handling techniques for safe lifting

All the items in the Large Maths Outreach and Careers Kit apart from the Penrose Tiles were manufactured by Richard Ellam www.lminteractive.co.uk. The Penrose Tiles were made by Edmund Harriss www.mathematicians.org.uk/eoh/

There is also an information sheet regarding a trebuchet included in this booklet. The trebuchet is not part of the Large Maths Outreach and Careers Kit; there are however good quality models which you can build yourself. The place where the IMA purchased their trebuchet from was www.catapultkits.com

Contents

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Double Pendulum

What is it?

The double pendulum is made of a simple pendulum, with another pendulum hanging freely from the first pendulum. The equations which describe the motion of a single pendulum are well defined so it could be easy to make the mistake of thinking that the double pendulum will move in a simple way as well ...

What is special about the double pendulum?

You will see that when you release the double pendulum it moves about in a wild and unpredictable way.

If you try and release the pendulum from one position and then repeat what you did, the pendulum is likely to move about in two very different ways.

This is because the double pendulum moves chaotically.

chaos

Systems which exhibit chaos generally have the following features:

1. They can be modelled or described using equations, so we aren't usually talking about randomness. *(The equations for the motion of a pendulum are well known.)*

2. Very small changes to the starting conditions will produce widely different results. This is popularly known as the butterfly effect. *(Release the pendulum twice and it is unlikely to do what you did the first time, however much you try.)*

3. They are unpredictable and unrepeatable, i.e. no two experiments produce the same results.

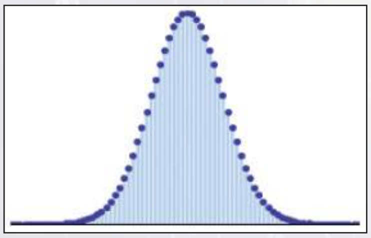
Probably one of the most famous chaotic systems is the weather system.
This is the reason that weather forecasters only predict the weather for the next few days.

Galton Board

What is it?

The Galton Board (also known as a quincunx or bean machine) was invented by Sir Francis Galton to demonstrate some important concepts in probability.

The board consists of a triangular array of pegs. Balls are dropped in at the top and bounce downwards, left and right before being collected at the bottom. If you drop enough balls in, then they will form a bell shaped curve like the one below.



What is special about it?

The bell shaped curve is an approximation of one of the most famous curves in history: the normal distribution curve. A surprising number of things follow this curve and it crops up in many areas including engineering, science and psychology.

One simple example is if you take a random group of women, measure their shoe sizes and plot how many times each shoe size appears.

Not many women will have really small feet (the left side of the graph) not many women will have huge feet (the right side of the graph) and most women will have sizes between 4 and 8 (the middle of the graph).

Why does the Galton Board produce this graph?

Each time a ball hits one of the pegs, it bounces either left or right with an approximately equal chance. This is repeated each time the ball passes a row of pegs until the ball rests at the bottom.

If you have studied A level statistics then you will be able to see that the distribution of how many balls collect in each bin at the bottom will be binomial.

When you have a large number of pegs the binomial distribution is approximately the same as the normal distribution. This is why the bell shaped normal curve starts to form.

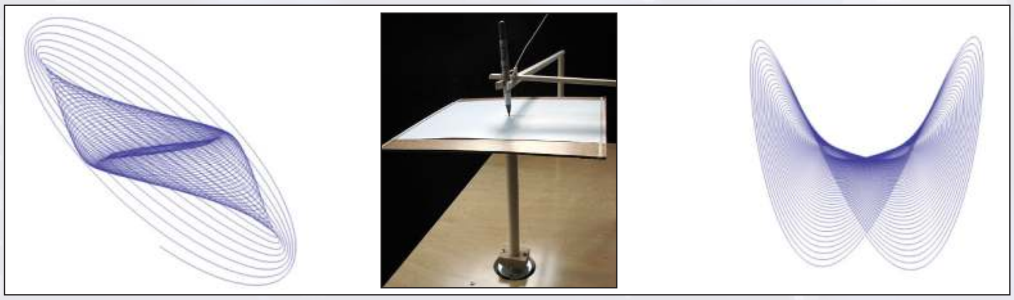
Harmonograph

What is it?

You might think that a pendulum is a simple thing, swinging back and forth in a predictable way, and if it had a pen attached to it, then it would produce nothing more than a straight line.

If, however, you attach a second pendulum to your paper, swinging at right angles to the first pendulum then you start to produce beautiful and surprising drawings. This is exactly what the Victorians did when they invented harmonographs: Simple machines which using two, three or four pendulums, create a breathtaking array of drawings.

The harmonograph on display uses two pendulums.



How does it work?

It is possible to describe the motion of two damped pendulums with the following parametric equations. One pendulum is moving in the x direction, the other pendulum is moving in the y direction.

$x(t)=A_1\sin(tf_1+p_1)e^{-d_1t}$ $\qquad$ $y(t)=A_2\sin(tf_2+p_2)e^{-d_2t}$

A_1 and A_2 are the amplitudes of the two pendulums.

d_1 and d_2 are the damping terms (damping is caused by effects such as air resistance and the bigger the damping terms, the quicker the pendulums will slow down and come to a halt).

p_1 and p_2 are phase factors between 0 and 2π. (This decides how out of phase the two pendulums are with each other.)

f_1 and f_2 are the frequencies of the two pendulums.
Without the damping (i.e. without any air resistance and friction) these equations describe what are called Lissajous curves.

The curves are very sensitive to the ratio $\frac{f_1}{f_2}$. If the ratio is 1, then the figure will be an ellipse. The curve will only be closed if $\frac{f_1}{f_2}$ is rational.

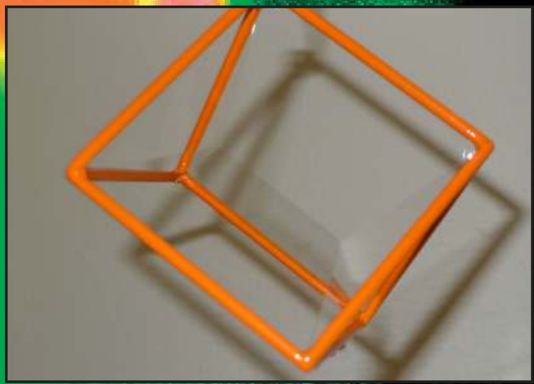
How can I find out more?

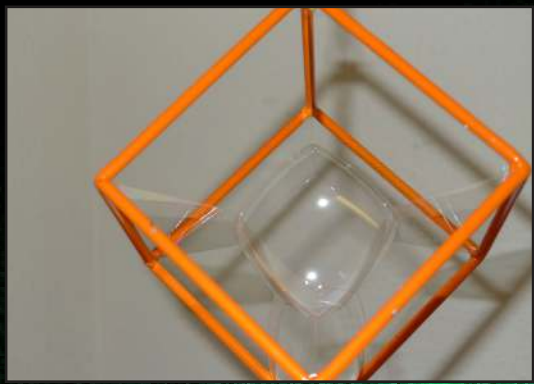
Have a look at www.walkingrandomly.com/?p=151 where you can download a program which simulates the harmonograph.

Soap Bubble Wires

What is it?

Dip the wire frames into the soapy liquid and when you put them gently out you should be left with a variety of surfaces which have formed inside the wire.





What is special about these surfaces?

Soap bubbles like to minimize their energy and their surface area. If you look at a soap bubble in the air it will be a perfect sphere. This is because a sphere is the shape with the smallest surface area which encloses a given amount of air.

In the case of the wire frames the soap bubbles form a surface which has the wire as its boundary, but also has the minimum surface area. This particular surface is then called a minimal surface.

Minimal surfaces are important to mathematicians, but also to engineers and designers. For example, designers sometimes want structures where stress is distributed as evenly as possible, so they choose minimal surfaces.

Top Tip: We have found that making a mixture of roughly 6% washing up liquid and 94% water makes a good solution. Stir it gently and you can prepare it well in advance of your session.

Travelling Salesman Problem

What is it?

The travelling salesman problem is a classic problem which has been studied by mathematicians for nearly a hundred years.

The problem is simple. A travelling salesman has a number of towns he wants to visit, before finishing where he started out. What is the shortest path he can take?

One option would be to try out all possible routes and then compare them to see which is the shortest. This quickly becomes too difficult, with 10 cities there are 3,628,800 possible routes, for 100 cities it would take a computer 40 million years to compare all the routes!

How can you find a good route?

Finding an exact solution is much too time consuming, even for a powerful computer, therefore mathematicians have designed algorithms to find approximate solutions which are close to the shortest solution.

One of the first algorithms which was developed was the Nearest Neighbour Algorithm. This requires you to move between the cities, always choosing to move next to the city which is closest and also hasn't already been visited yet. This algorithm sometimes produces a good result; however, it also sometimes produces the longest route!

Therefore mathematicians have been on the hunt ever since for better and better algorithms. Approaches have included using genetic algorithms where different routes are 'bred together', the bad results are thrown away until eventually a much better route is developed.

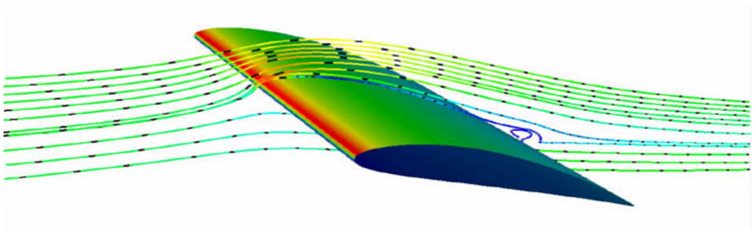
Why is the Travelling Salesman Problem Important?

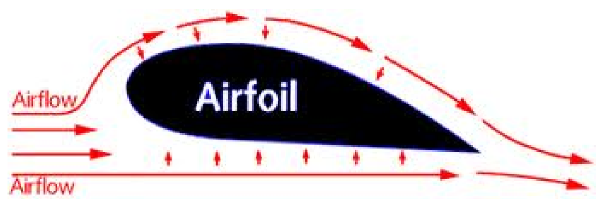
The Travelling Salesman Problem crops up all over the place - amongst other things, it is important in timetabling problems, in designing traffic networks and in designing computer networks.

Aerodynamics

How does a jumbo jet fly?

The secret is in the design of the wings.





The lift is caused by the fact that the air flows faster on the upper surface of the wing compared to the lower surface.
To understand lift, you need to know about **pressure** p and **density,** ρ .

What is pressure p ?

Air molecules are in a state of random motion. When they bounce off a surface they produce a force. Pressure is the force that the air molecule impacts would produce per unit area of surface.

What is density ρ ?

ρ is the mass of air in a unit volume. ρ depends on the number of air molecules contained within a unit volume.

Bernouilli's Equation

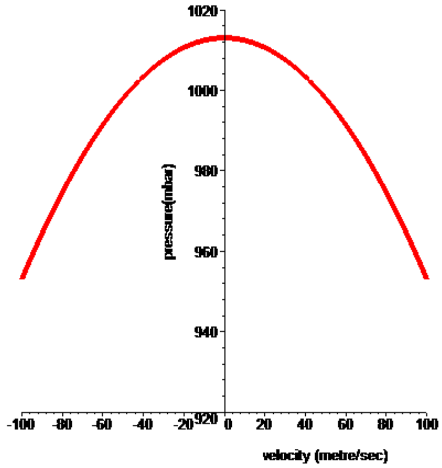
The relationship between pressure and air speed is:

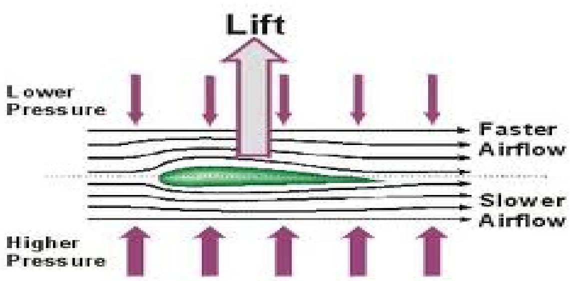
Pressure + $\frac{1}{2}$ density x (speed)^2 is constant

This is Bernouilli's Equation and written mathematically as

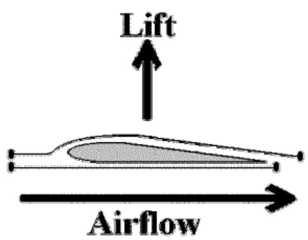
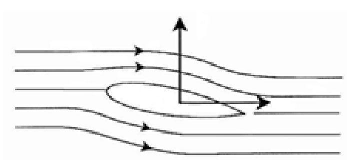
$$p + \frac{1}{2}\rho v^2 = \text{Constant}$$

Hence if v is high, p is low
and if v is low, p is high .

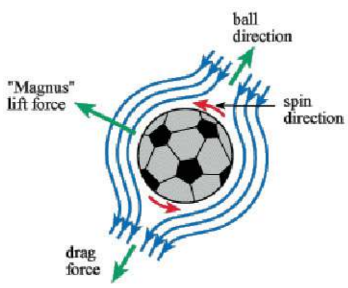




Both the shape of the aerofoil and the angle of incidence to the flow are important.

(i) Flow past an asymmetric aerofoil 	(ii) flow past a symmetric aerofoil inclined to horizontal 

How do cricketers or footballers make balls swerve?



The movement is generated by the introduction
of circulation through spin and using the Bernouilli effect.

Trebuchet

What is it?

The trebuchet is a medieval energy weapon...but it uses gravitational potential energy. It was used during the middle ages as a siege weapon to batter down castle walls by hurling heavy stones at them. It was also very versatile and could be used to throw incendiary bombs or even dead animals in an early example of biological warfare.

How does it work?

It is built like a very unbalanced set of scales. On one side is a heavy mass called the counterweight. On the other side is a missile, which would have been a heavy stone, but fitted in a sling which effectively lengthens the other arm, but without adding much mass. This is important in how far the machine will hurl a rock. This rock is much lighter than the counterweight. It starts with the counterweight being raised by human effort. In this position, it has high gravitational potential energy. When the weight is released, it falls and so raises the arm attached to the sling. The arm flies up to a roughly vertical position where the sling releases the rock which flies at high speed toward the enemy castle walls.

The trebuchet follows the Law of Conservation of Energy, so as the counterweight loses potential energy, the rock gains kinetic energy. The aim of the siege engine designer is to build a machine that can transfer as much of the potential energy of the counterweight to kinetic energy of the rock.

Of course, medieval siege engine builders had to work all this out by trial and error, by building lots of prototypes. By analysing the trebuchet mathematically, you can make a mathematical model... and with this, you can develop a much better siege engine.

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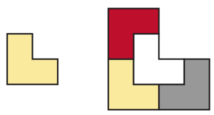
The Penrose Tiles

The Penrose Tiling

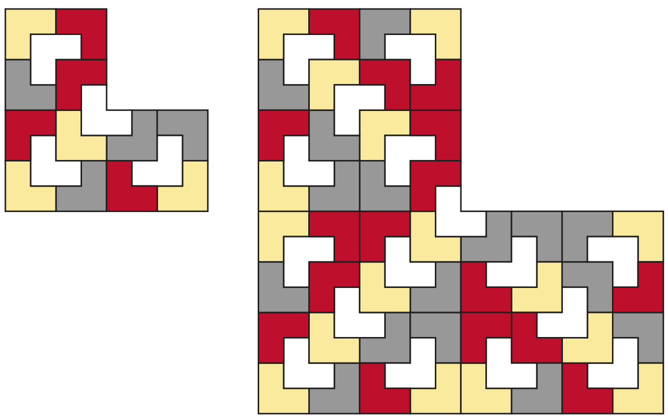
These two shapes have a remarkable property. They can tile the plane but not periodically. In other words the tiling of the plane they produce is not made of a single patch of tiles repeated. We hope here to show you the proof of this fact, remember a proof is in many ways just a simple explanation. To do that we need to do two things. Firstly we need to show that the tiles can in fact tile the plane. Secondly we need to show that no periodic tiling is possible. To show that they tile the plane we need a new construction called a substitution rule.

Substitution Rule

Think about this L-shaped tile, we can double it in size and then cut this larger shape into four copies of the original:



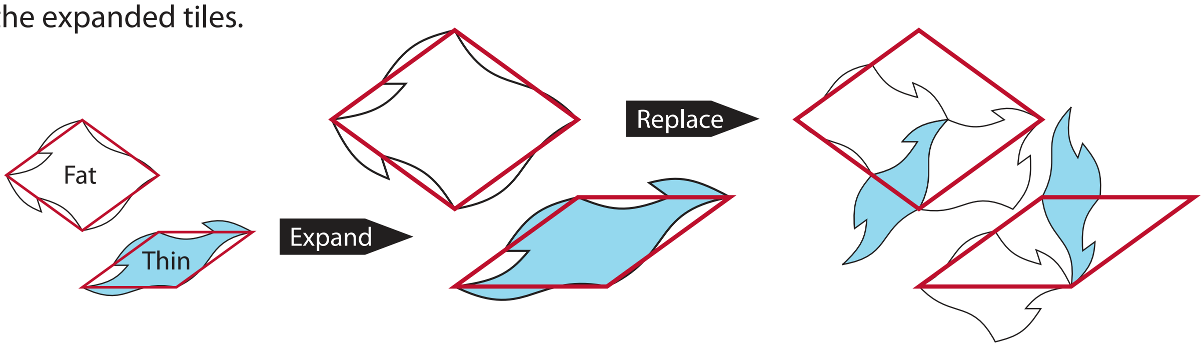
We can now repeat...



This is a simple example of a substitution rule. In general a substitution rule takes a patch of tiling, expands it and then replaces the larger tiles by patches of the original ones. So in this case we take the L-shaped tile, expand it by a factor of two and then replace it by four copies.

Penrose Substitution

The substitution rule for the Penrose tiles is a little more complicated. In this case we expand the tiles by the golden ratio: $(1+\sqrt{5})/2$, and then replace the larger tiles by the following two patches. Note that these patches of tiles fit together in exactly the same way (though with different edges) as the expanded tiles.



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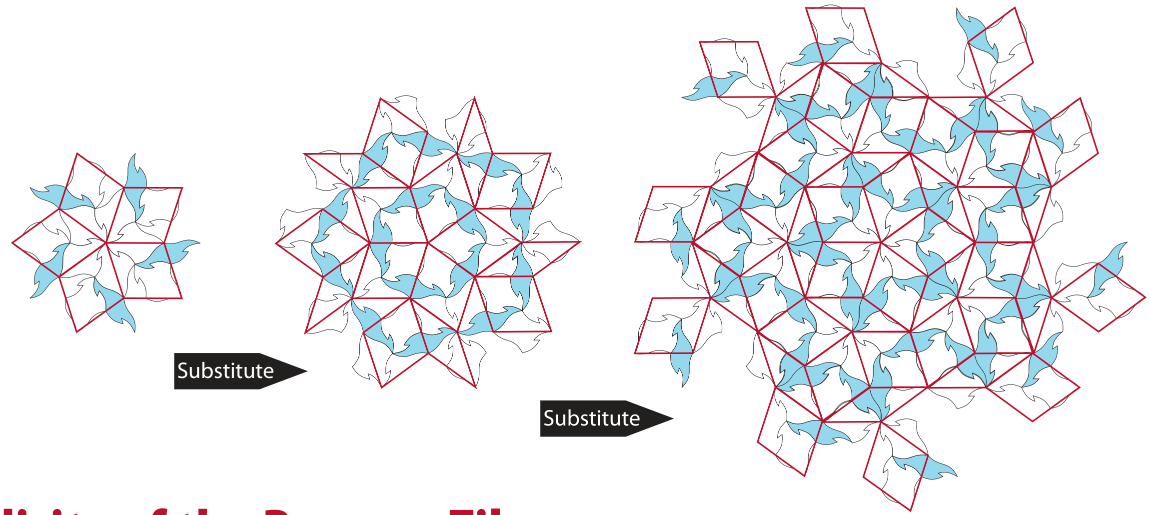
Edmund Harriss, Uwe Grimm, Richard Henry, John Hunton, Vinay Kathotia, Jeroen Lamb

Notice that these two patches can fit together in exactly the same way as the original two tiles. We can therefore think of them as larger versions of the original tiles. We can repeat this to get larger and larger patches of Penrose tiles:



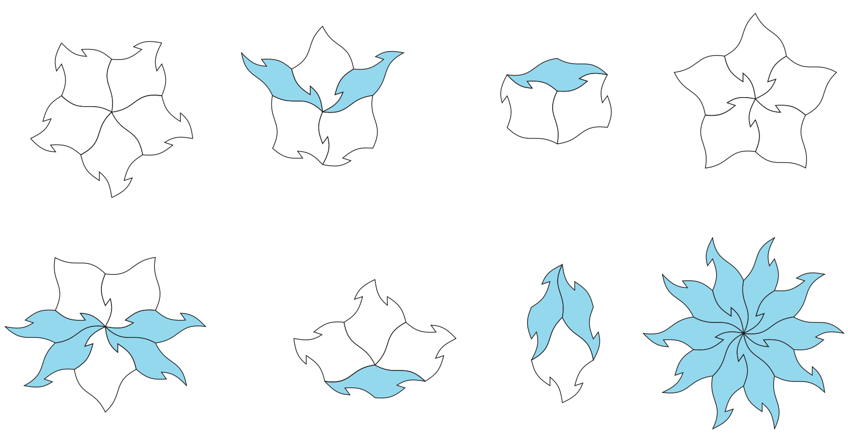
Aperiodicity of the Penrose Tiles

We now know that the Penrose tiles can fill the plane. Can this tiling be periodic? What happens when we start with a single fat rhomb and start to apply the rule. After one application we have 2 fat rhombs and 1 thin rhomb. At each stage we replace the fat rhombs again by 2 fat and 1 thin, and the thin rhombs by 1 fat and 1 thin. So the numbers of fat and thin rhombs (thin, fat) go as follows:

(0,1), (1,2), (3,5),(8, 13),(21, 34), (55,89), (144, 233)...

This sequence might be familiar to you, these are the Fibonacci numbers, where each number is the sum of the two before. The ratio of a Fibonacci number to its predecessor gets closer and closer to the golden ratio $(1+\sqrt{5})/2:1$ as the numbers get larger. This is therefore the ratio of fat rhombs to thin rhombs in a tiling generated by the substitution rule (as this involves applying the substitution infinitly often). This is an irrational number, one that cannot be written as a fraction. Think of a periodic tiling: it has one patch repeated, so the ratio of the tiles must be the same as the ratio in the patch. Thus the tiling given by this substitution rule cannot be periodic.

So the Penrose tiles can tile, in a non-periodic way. Is there another way? No. Lets think about how they can fit round a point, some are shown below (we have done a little of the work for you and ruled out a few that cannot continue). Can you show that these patches will either occur in the substitution tiling, or cannot be continued? Try it using the shapes provided.



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How do shapes fill space?

What shapes fill the plane?

A History of Aperiodicity

In the early 1960's the mathematician Wang was thinking about the question:

If you are given a set of shapes of tiles, can you use them to tile the whole plane?

Wang thought that there should be an algorithm or computer program that would be able to decide, for any set of shapes of tiles you gave it, whether or not you could use them to fill the plane. He was assuming however that if you could fill the plane with your tiles, then it would have to be by using them to form a patch that could be endlessly repeated.

If it had been true that any tiling of the plane had to be the repetion of some basic patch, then Wang's algorithm would have gone as follows:

- Take one tile shape. Does it form a patch that can be made to repeat?
  If it does, you've got your answer. If not, test each other shape.
- If all the shapes fail, try all possibilities of 2 shapes put together.
  If one of them forms a patch that repeats then you're finished.
  If not, test all ways of putting 3 shapes together, and so on.

Either – you eventually get to a patch that can be made to repeat:
you can tile the plane with these shapes of tiles;
Or – you get to a point where you can't fit any more tiles together:
you can't tile the plane with these shapes of tiles.

However, in 1966 Wang's student Berger showed there was a problem with this: he discovered a set of tiles that did tile the plane, but in a way that never repeated. In fact, the tiles could never be put together to form a patch that could be repeated: the tiles have to be continually put together in different configurations. Any algorithm of the sort outlined above would never finish. Berger had found the first example of an aperiodic tiling in 1966. He needed 20,426 different tiles to make it. Once the first example had been found, others quickly found aperiodic tilings using smaller sets of tiles. By 1971 Raphael Robinson had a tiling using just 6 tiles, but a few years later Roger Penrose had his, using just 2 tiles.

Less than 10 years later the patterns these tiles made had been found in nature as the positions of atoms in metal alloys. The 3 dimensional analogues of these 2-d tilings are now understood as key shapes in the structure of viruses and are being used to understand and predict virus evolution.

The quest for a monotile. Penrose's tiling is made of just two shapes of tile. At the moment, no one knows if there is a single shape of tile that tiles the plane but only aperiodically. Can you find one, or prove that there can be no such tile?

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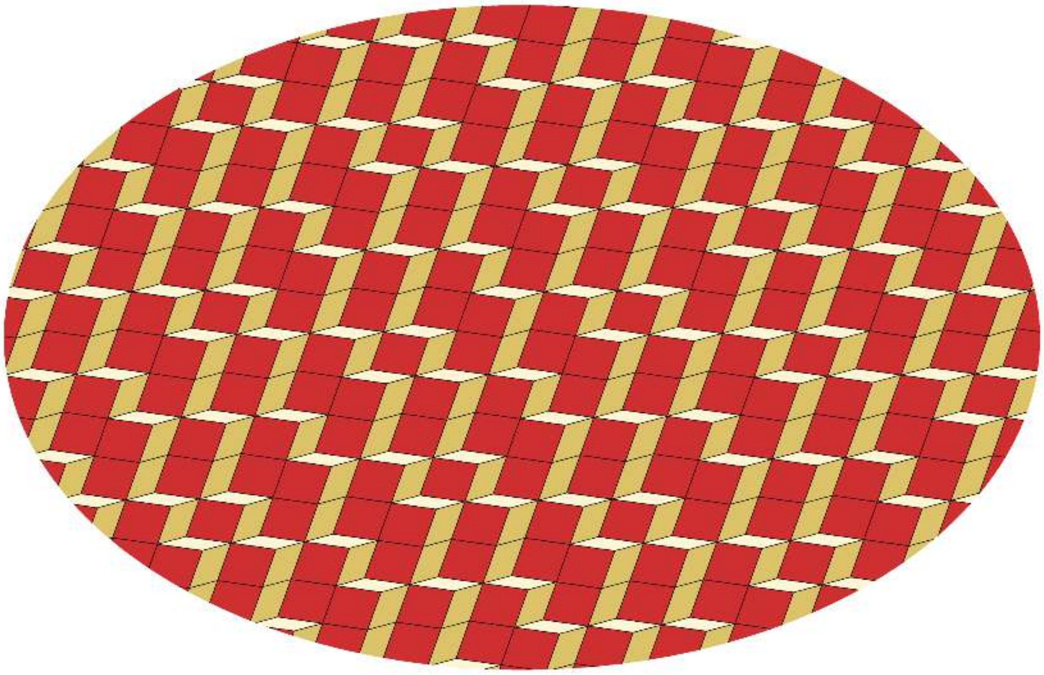
Non-periodic slices

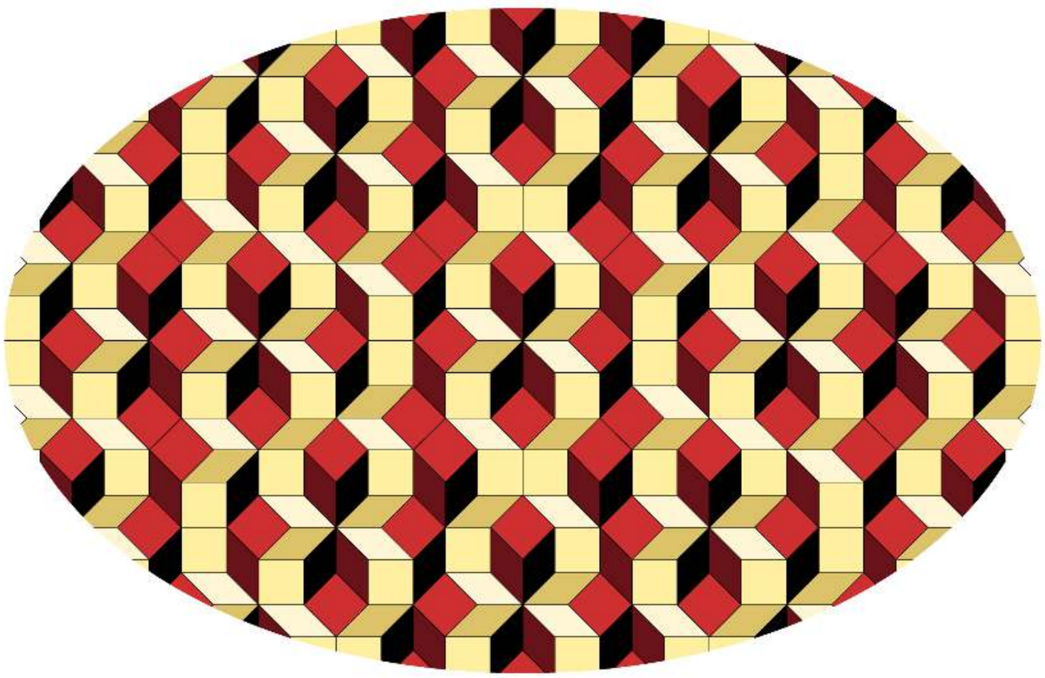
How can we easily generate non-periodic tilings? Here is a method that has proved to be very useful. Take a look at this:



Is this a picture of a stack of blocks, or is it a tiling made from 3 different shapes of tile?

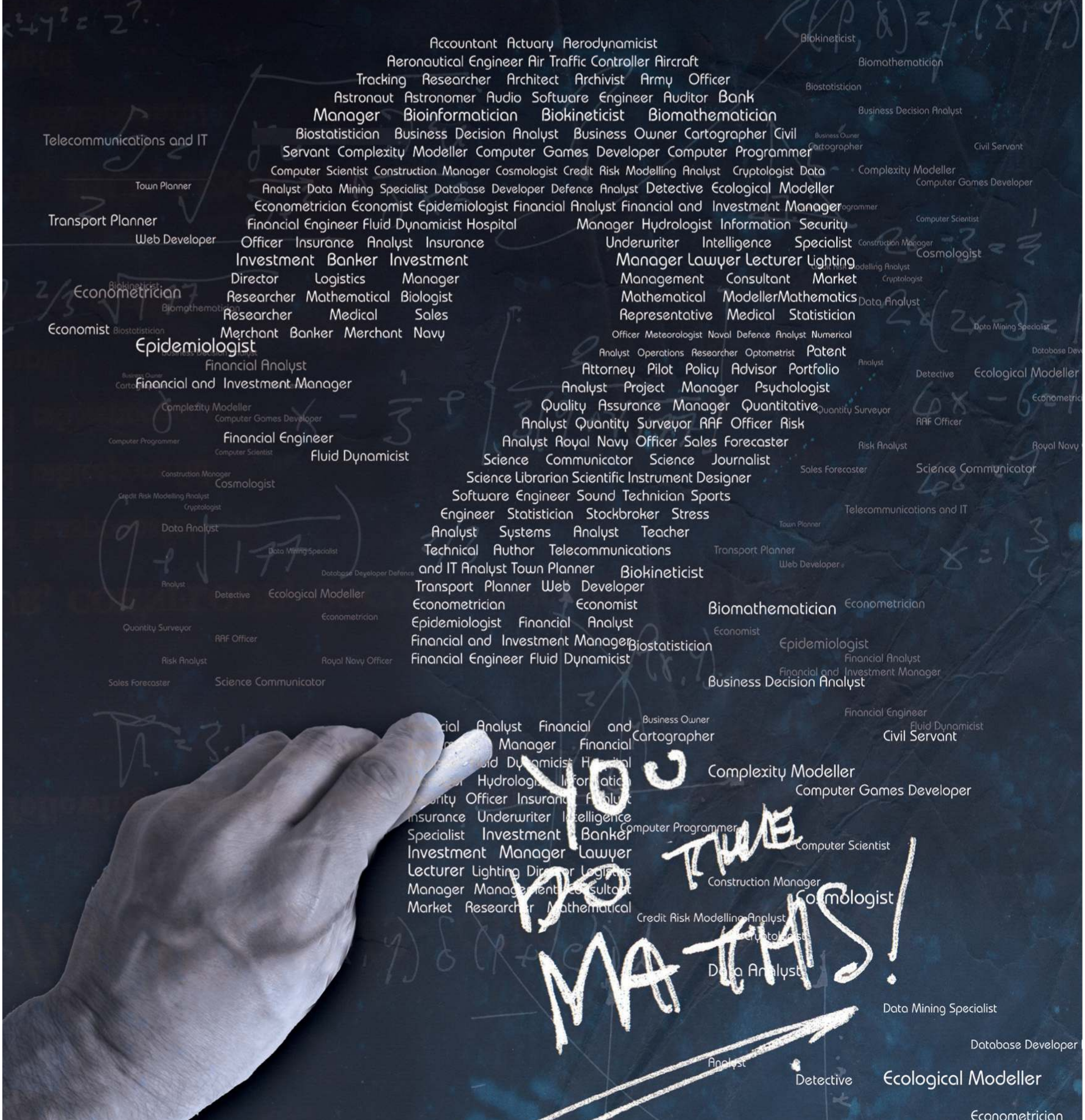
Well it's both! One way to create unlimited examples of aperiodic tilings is to take a periodic tiling of a higher dimensional space - the picture above used a tiling of 3d space by cubes - and then taking a slice through it. So long as the slice is at an irrational slope with respect to the sides of the cubes the result is a non-periodic pattern of tiles.

Here is another example:



this time a 2d slice through a tessellation of 4d by 4 dimensional cubes. The Penrose tiling can be made as a slice through 5d space.

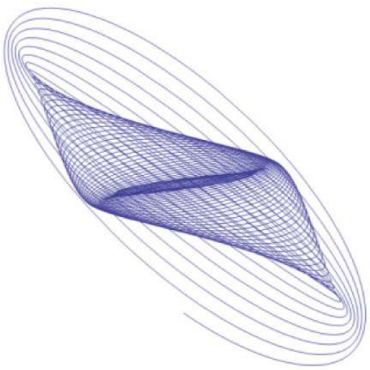
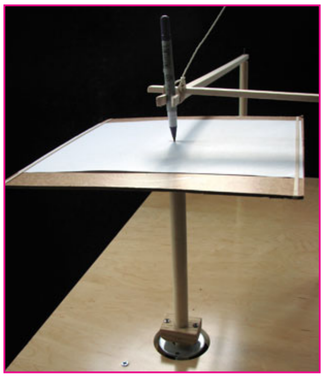
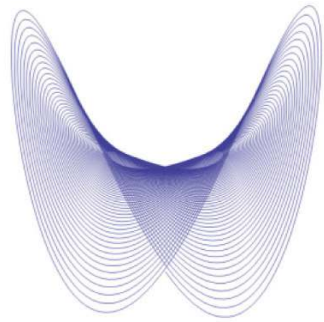
Which subject at degree level gives you access to a thousand different careers?



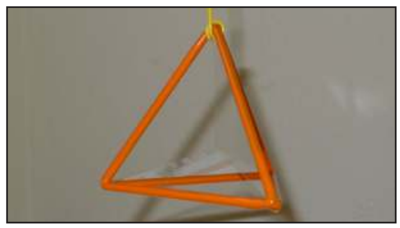
Visit www.mathscareers.org.uk for more exciting information!










We would be grateful if you could please photocopy this form and send it back to the IMA

1) Which item(s) did you borrow/use?

....

....

2) Describe the event you used the item(s) at?

....

....

3) Approximately how many people interacted with the items during the time that you borrowed them for?

....

....

4) What was successful about using the item(s)?

....

....

5) Was there anything which you think could have been improved?

....

....

6) Was the shipping to your institution trouble free?

....

....

7) Are any repairs or maintenance required on the item(s) used e.g. Making the items ready for the next user?

....

....

8) Any other comments?

....

....

9) Your name and contact details

....

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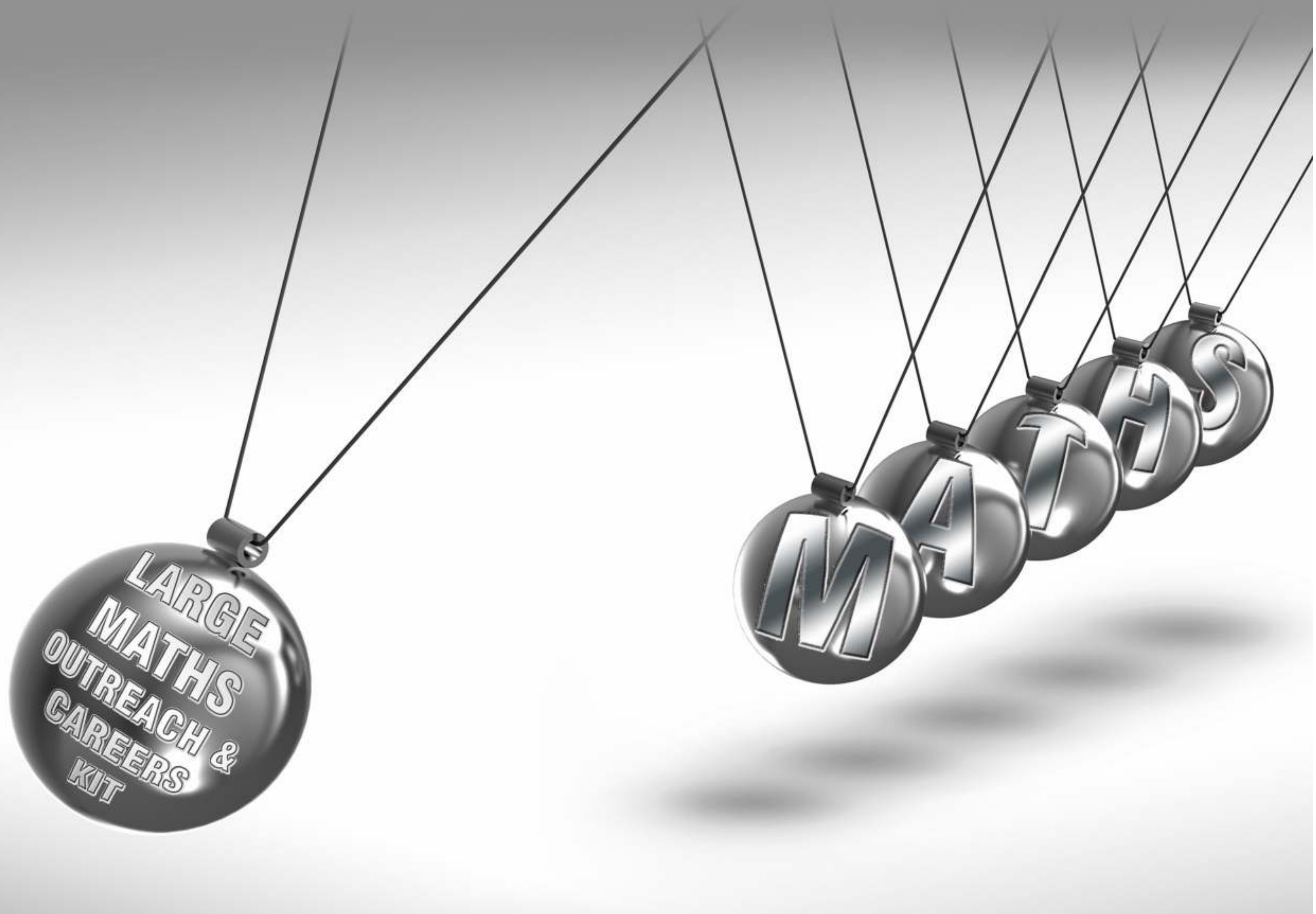
We welcome your feedback

Institute of Mathematics and its Applications
Catherine Richards House, 16 Nelson Street, Southend-on-Sea, Essex SS1 1EF



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